• How could you calculate the Fermi surface of a metal?
• How could you measure the Fermi surface of a metal?
• What are Shubnikov-de Haas oscillations and how are they related to the Quantum Hall effect?

How could you calculate the Fermi surface of a metal?

First of all you have to know the fermienergy. The next step is to calculate the energy versus k relationship with some bandstructurmethod. Some examples are the following:

• Tight binding
• Plane wave Method
• Augumented plane wave method
• Density functional theory
• Hartree - Fock

Some methods are simple and don’t give good results like the planewave method or tight binding, other methods are more sophisticated like the density functional theory or Hartree Fock. The Fermi surface is a plot of constant energy with \( E = E_f \). Figure 1 shows the Fermi surface of copper and the corresponding energy versus k relationship and density of states.

Abbildung 1: Metall: copper, top: Fermi surface, left: Bandstructur, right: Density of states
How could you measure the Fermi surface of a metal?

The period of the de Haas - van Alphen oscillations is indirectly proportional to the cross section of the Fermi surface with the Landau cylinders (equation 1). Figure 2 shows how they look like. If the magnetic field is pointing in the z direction the k states in the xy plane are restricted to circles and in the z direction nothing happens. That’s why you get cylinders in 3 dimensions. If you apply a magnetic field to your metal in one direction and change the magnitude of the field, the Landau cylinders start moving through the Fermi surface, since the spacing between them is proportional to B. The result of these are oscillations in all properties (everything that depends on the density of states like the internal energy). In this special case of the de Haas - van Alphen oscillations it’s the magnetization. Figure 3 shows the current which is flowing if you apply a magnetic field in the 111 or the 100 direction for this metal. Now you have to apply the magnetic field in lots of different directions to get information about the whole Fermi surface. It can happen that more than one region of the Fermi surface is parallel to the Landau cylinders. In this case you get a beating pattern. Which is a modulated basic oscillation. You could fouriertransform it and that will give you the frequency components and the amplitudes of the oscillations. You can calculate the period out of the frequencies and use equation 1 to match them with the corresponding cross section.

\[ S \left( \frac{1}{B_{n+1}} - \frac{1}{B_n} \right) = \frac{4\pi^2 e}{h} \]  

(1)

S... cross sectional area of the Fermi surface 
\[ \left[ \frac{1}{B_{n+1}} - \frac{1}{B_n} \right] \].....period
What are the Shubnikov-de Haas oscillations and how are they related to the Quantum Hall effect?

The Shubnikov-de Haas oscillations are oscillations of the resistivity. They are shown in figure 4. Figure 5 shows the energy versus position relationship. This is very important to understand the oscillations. The picture on top shows how the normal E versus position would look like in a magnetic field without an applied current. The different energy levels are the different Landau levels. The curves bend up, because the electrons are confined to the sample, otherwise they would be able to leave the metal without performing work. \( \mu_R \) and \( \mu_L \) indicate the right and left going electron states. The electrons on this positions move along the edge. The electrons in the middle move in circles. If we apply a current in one direction one edge state gets more populated and the other one less. The result of this is a net current flow. Before that the current to the left was equal to the current going to the right. If the fermienergy is far away from the Landau levels, then there are just the states at the edge. And it is impossible to scatter electrons from the right going states into the left going states and vice versa. The distance between \( \mu_R \) and \( \mu_L \) is to large (phonons at roomtemperatur have a pretty low k value). There will be no resistivity. When we increase the magentic field the Landau levels move up in energy. And at some point they are close enough to the fermienergy and there will be a lot of states to scatter into. The result will be a large resistivity. This hole behaviour is shown in figure 5. To sum up: The Shubnikov-de Haas oscillations are oscillations of the resistivity in the direction of the applied current. Figure 6 shows a setup to measure the Hallconstant. In this figure the current is applied to the x direction. \( \rho_{xx} \) would be the one you talk about the Shubnikov-de Haas oscillations, and \( \rho_{xy} \) is the Hall resistivity. Figure 7 illustra-
Abbildung 4: Shubnikov-de Haas oscillations

The reason for the behaviour of the Hall resistivity is equation 2. It was derived by taking into account that the electron density can be written as integer (number of Landau levels \( n \) below the fermienergy) times the degeneracy of the Landau levels, when the fermienergy is between the Landau levels. Figure 7 shows how the Shubnikov-de Haas oscillations and the Quantum Hall effekt are related. The hall resistivity is constant when the fermienergy cuts only through the edge of the band. In this situation the Shubnikov-de Haas resistivity is zero (when the magnetic field is high enough). When the fermienergy gets closer to the Landau niveaus there will be an increase in both resistivities. There is a huge peak in \( \rho_x \) and \( \rho_{xy} \) increases linearly.

\[
\rho_{xy} = \frac{E_y}{J_x} = -\frac{B_z}{n e}
\]  

(2)
Quantum hall effect

On the plateaus, resistance goes to zero because there are no states to scatter into.

Abbildung 5: energy diagramm of the Shubnikov de-Haas oscillations a... without current, b,c....applied current but different B field

The Hall Effect (diffusive regime)

\[ R_{xx} = \frac{\rho_{xx} L_x}{L_y L_z} = \frac{V_x}{I_x} = \frac{E_x L_x}{j_x L_y L_z} \]

\[ \rho_{xx} = \frac{E_x}{j_x} \]

\[ \rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne} \]

multiply both sides by \( B_z \)

Abbildung 6: experimental setup for measuring the Hall constant
Abbildung 7: $\rho_{xx}$... Shubnikov-de Haas oscillations, $\rho_{xy}$... oscillations in the hall resistivity