Crystal momentum $\hbar\vec{k}$

$$i\hbar\frac{d}{dt}\langle A\rangle = \langle AH - HA\rangle$$

translation operator $T\psi(x) = \psi(x+a)$

<T> is a constant of motion for a perfect crystal

$$i\hbar\frac{d}{dt}\langle T\rangle = \langle TH_0 - H_0T\rangle = 0$$

Consider an external force in the x-direction

$$F_{ext} = -\frac{dU}{dx} \quad \Rightarrow \quad U = -F_{ext}x$$

$$i\hbar\frac{d}{dt}\langle T\rangle = \langle T(H_0 + U) - (H_0 + U)T\rangle$$

$$i\hbar\frac{d}{dt}\langle T\rangle = \langle T(H_0 - F_{ext}x) - (H_0 - F_{ext}x)T\rangle$$

$$i\hbar\frac{d}{dt}\langle T\rangle = \langle -TF_{ext}x + F_{ext}xT\rangle = \langle -F_{ext}(x+a)T + F_{ext}xT\rangle$$

Crystal momentum

$$i\hbar \frac{d}{dt}\left\langle T \right\rangle = \left\langle -F_{ext}\left(x+a\right)T + F_{ext}xT \right\rangle$$

$$i\hbar \frac{d}{dt}\left\langle T \right\rangle = \left\langle -F_{ext}aT \right\rangle = -F_{ext}a\left\langle T \right\rangle$$

$$\left\langle T \right\rangle = \left\langle e^{-ikx}u_k(x)\middle| T \middle| e^{ikx}u_k(x)\right\rangle = \left\langle e^{-ikx}u_k(x)\middle| e^{ik(x+a)}u_k(x+a)\right\rangle$$

$$\left\langle T \right\rangle = e^{ika}\left\langle e^{-ikx}u_k(x)\middle| e^{ikx}u_k(x)\right\rangle = e^{ika}$$

$$i\hbar \frac{d}{dt}e^{ika} = -F_{ext}ae^{ika}$$

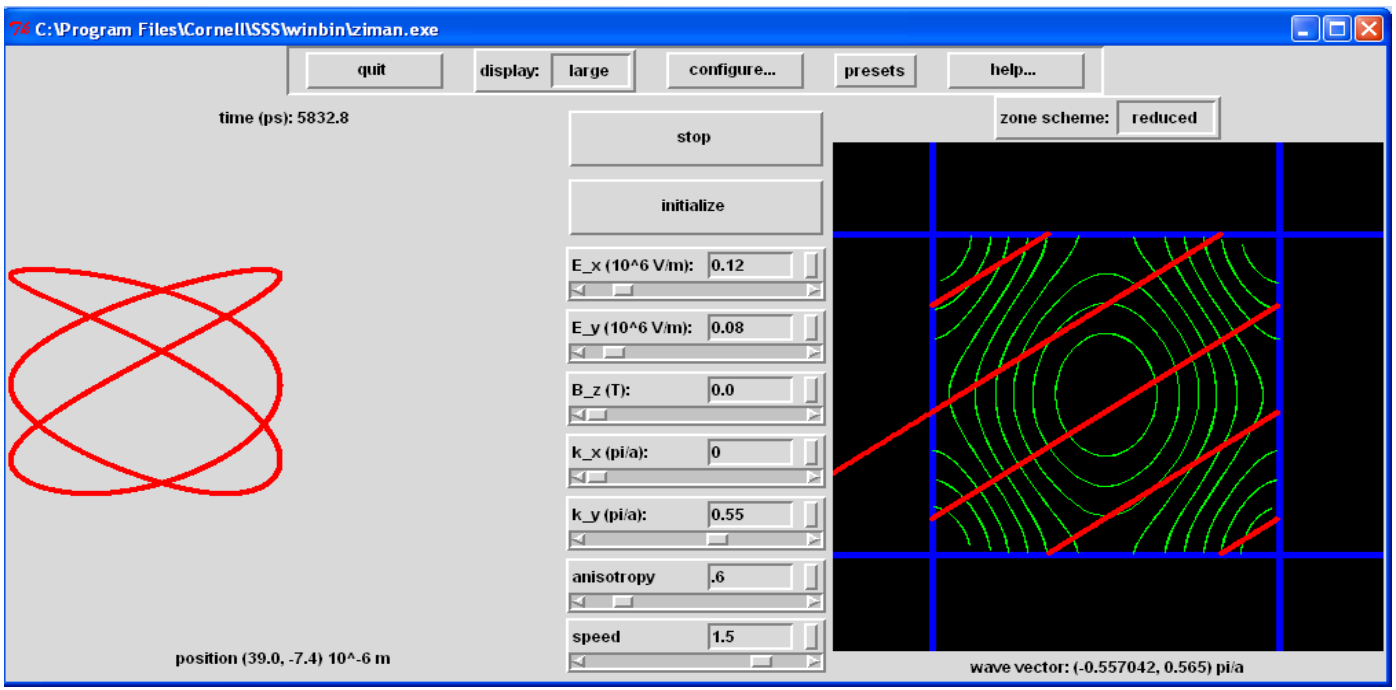
$$i\hbar(ia)\frac{dk}{dt}e^{ika} = -F_{ext}ae^{ika}$$

$$\hbar \frac{dk}{dt} = F_{ext}$$

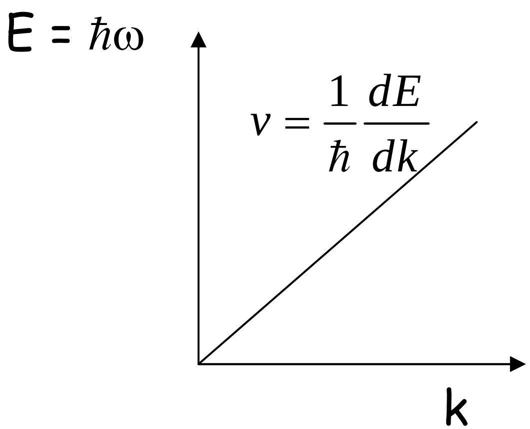
Crystal momentum

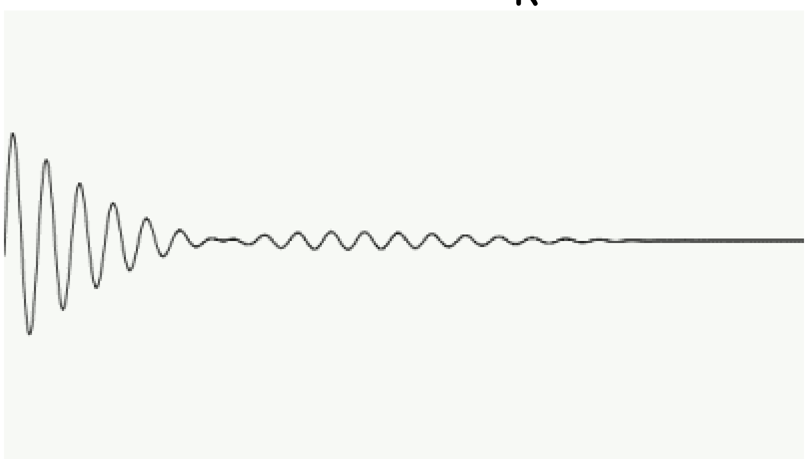
$$\hbar \frac{d\vec{k}}{dt} = \vec{F}_{ext} \qquad \frac{d\vec{p}}{dt} = \vec{F}_{total}$$

$$\hbar\vec{k} \neq \vec{p}$$



Group velocity





v_g is the velocity of a wave packet

$\Delta x \Delta k \sim 1$

$$v_g = \frac{1}{\hbar}\frac{dE}{dk}$$

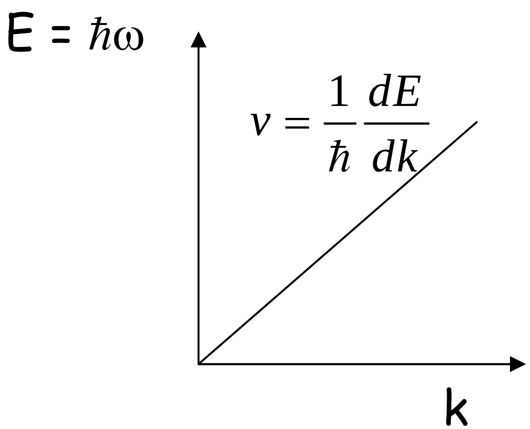
$$\frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dkdt} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}$$

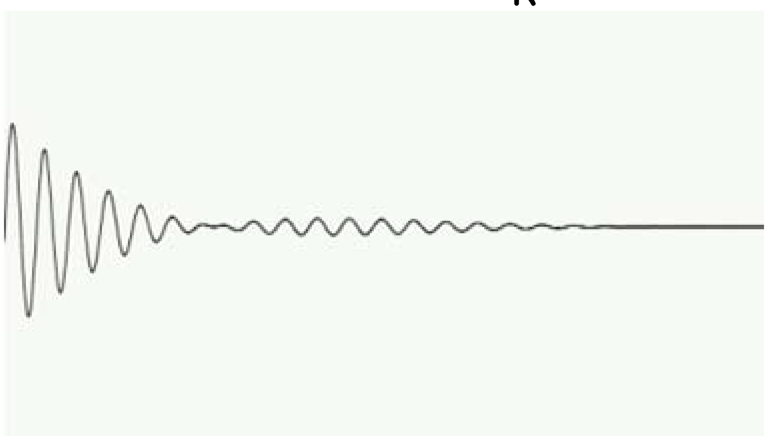
$$\frac{dv_g}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F_{ext}$$

$$F_{ext} = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}\frac{dv_g}{dt} = m^* a_g$$

Particles in a semiconductor can be thought of as free particles with an effective mass.

Group velocity





v_g is the velocity of a wave packet
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$$a_g = \frac{dv_g}{dt} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F_{ext}$$

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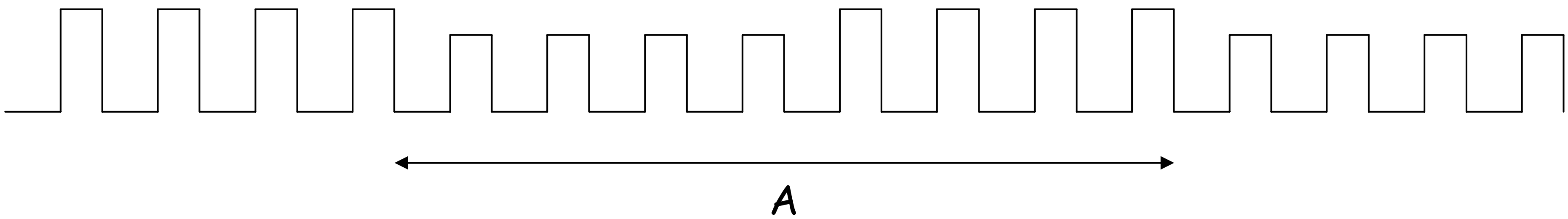
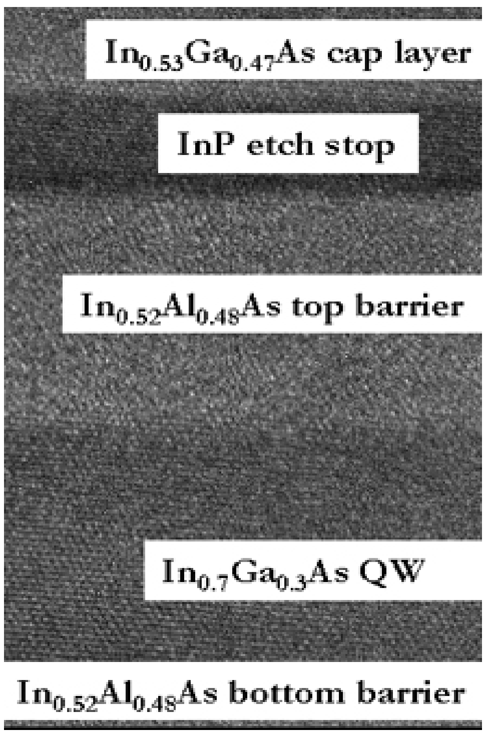
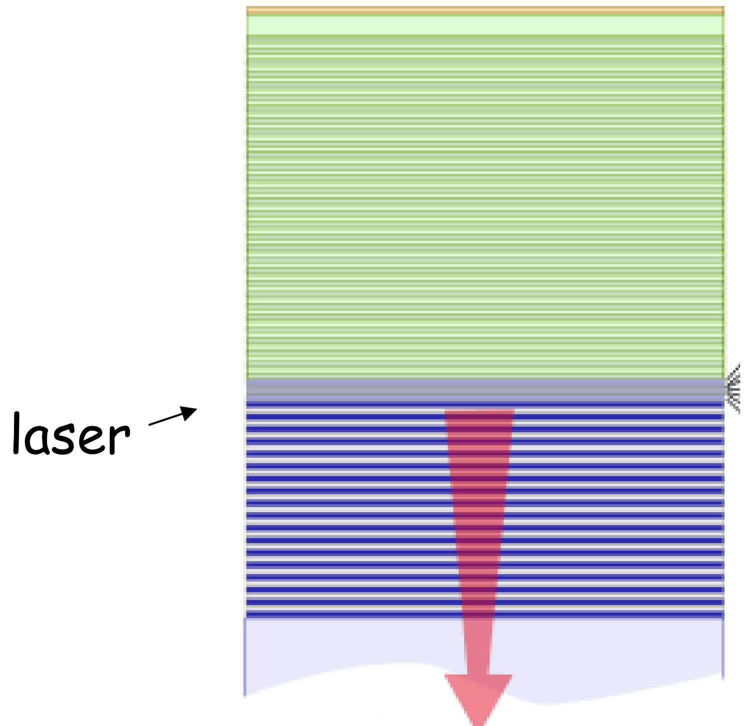
Particles in a semiconductor can be thought of as free particles with an effective mass.

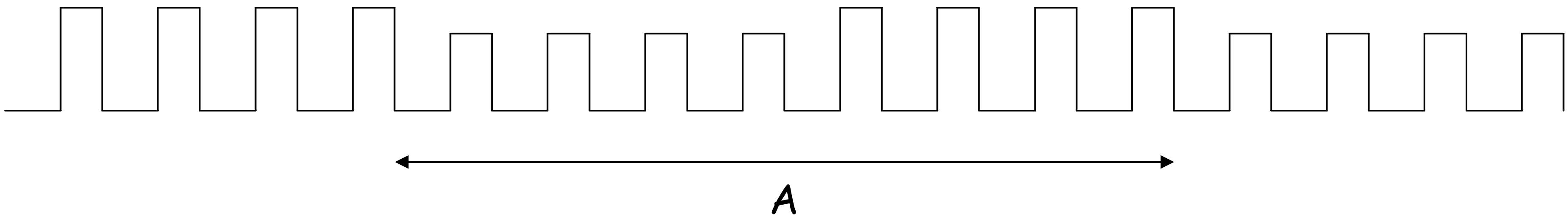
Mini bands in superlattices







Bloch oscillations



$$\varepsilon(k) = -\varepsilon_0 \cos(kA)$$

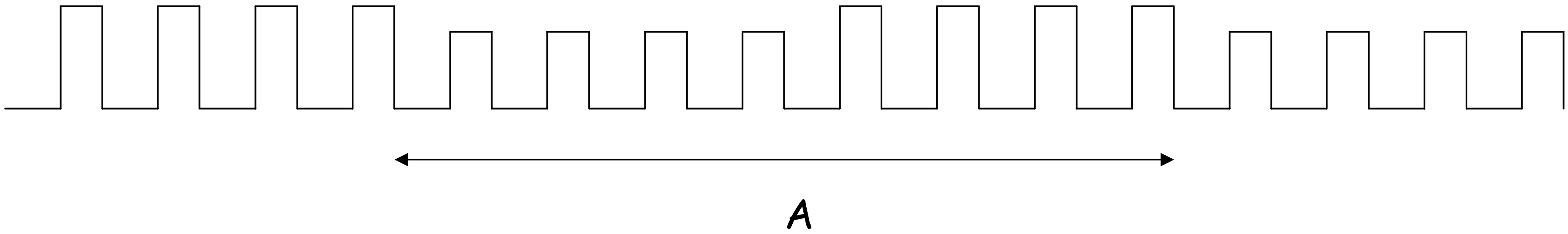
$$v(k) = \frac{1}{\hbar}\frac{d\varepsilon}{dk} = \frac{A\varepsilon_0}{\hbar}\sin(kA)$$

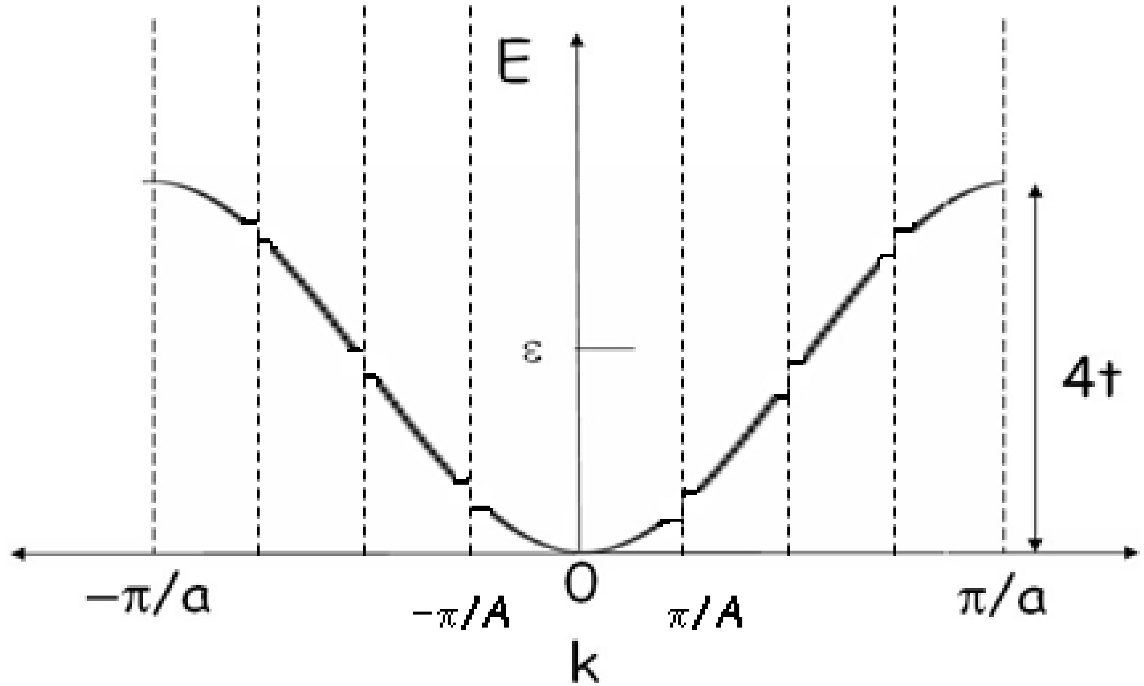
$$x = \int v\,dt = \int v(k)\frac{dt}{dk}dk \qquad -eE = \hbar\frac{dk}{dt} \quad \Rightarrow \quad k = -\frac{eEt}{\hbar}$$

$$x = \int \frac{-\hbar}{eE}\frac{A\varepsilon_0}{\hbar}\sin(kA)dk = \frac{\varepsilon_0}{eE}\cos(kA) = \frac{\varepsilon_0}{eE}\cos\left(\frac{eEAt}{\hbar}\right)$$

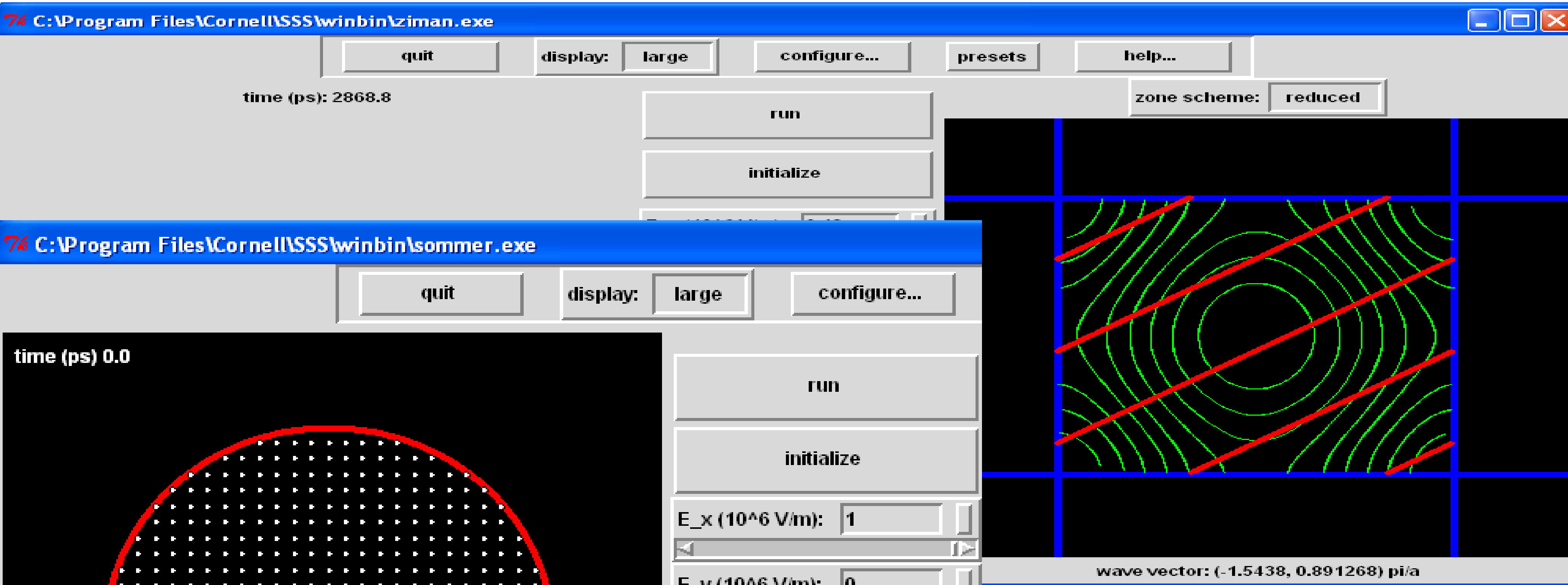
The Brillouin zone is much smaller for a superlattice. This makes the Bloch oscillation frequency $\omega_B = eEA/\hbar$ higher.

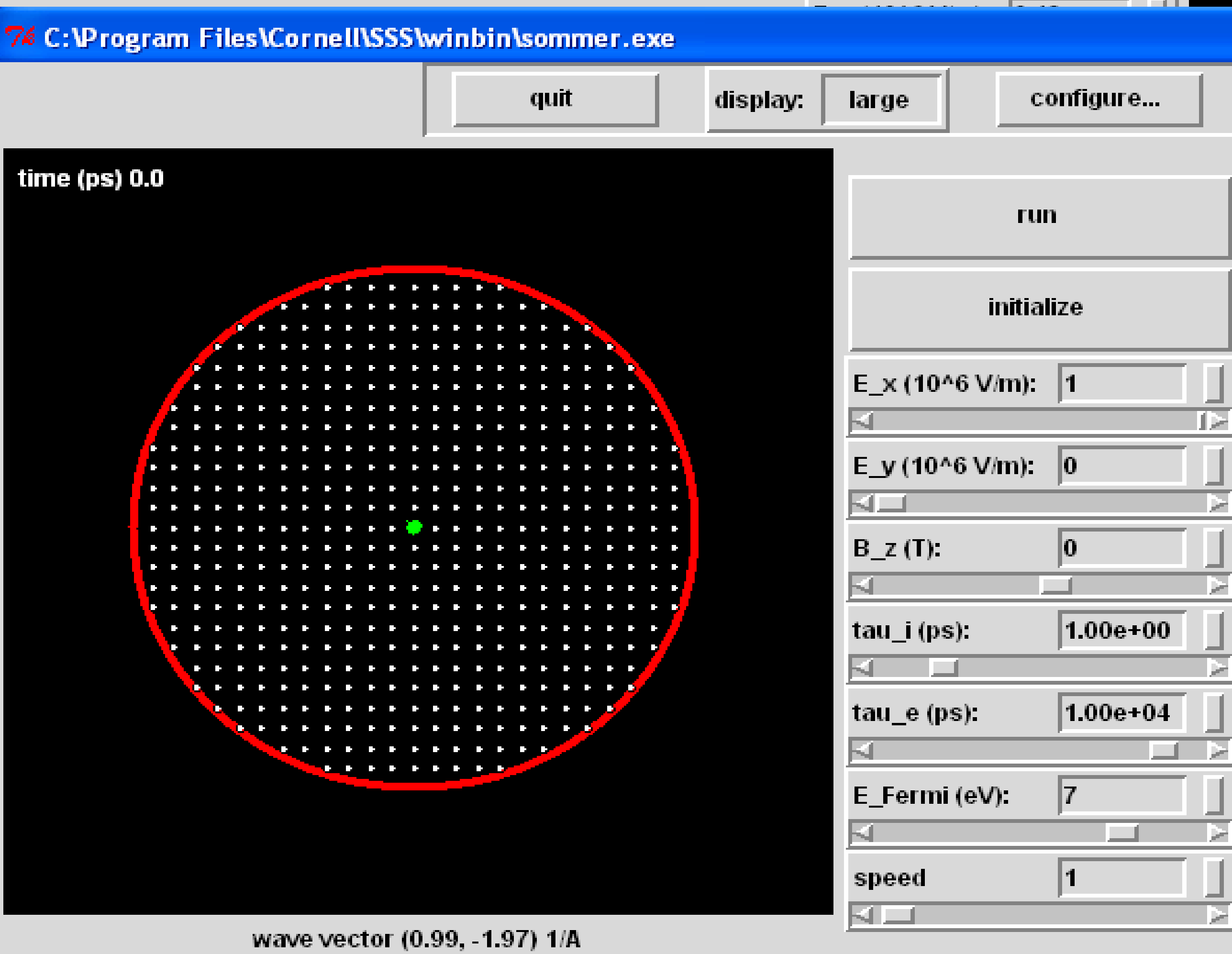
Mini bands in superlattices





Bloch oscillations





$$\omega_B = eEA/\hbar$$

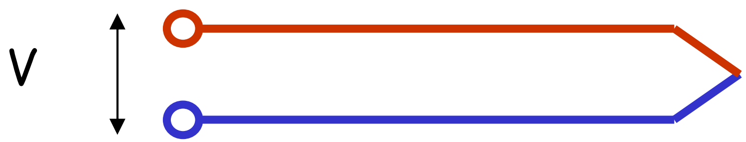
Thermoelectric effects

Seebeck effect:
A thermal gradient causes a thermal current to flow. This results in a voltage which sends the low entropy charge carriers back to the hot end.

$$\vec{E} = Q\nabla T$$

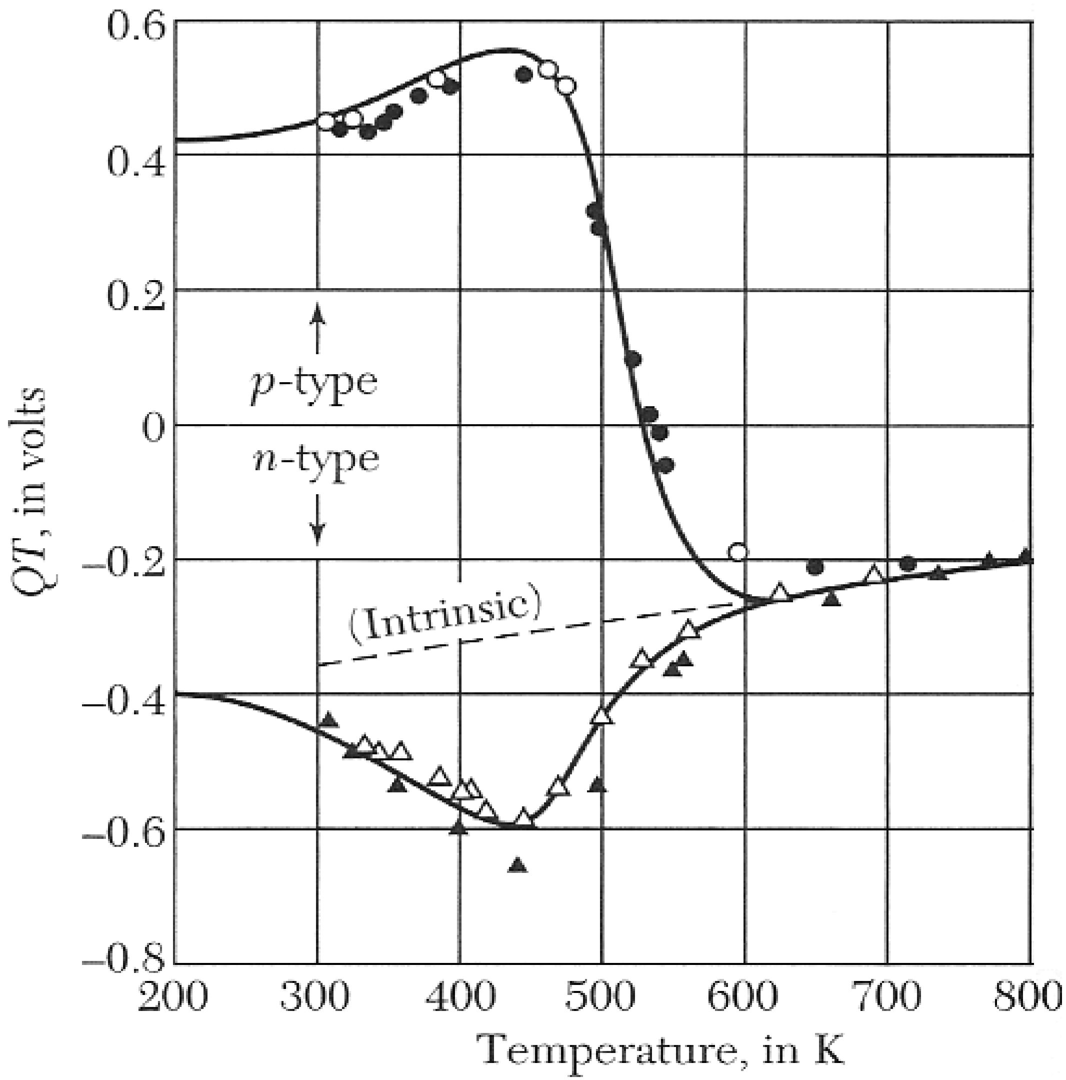
Q is the absolute thermal power. The sign of the voltage is the same as the sign of the charge carriers.

The Seebeck effect can be used to make a thermometer.



Thermocouple

Thermoelectric effects



Thermoelectric effects

Electrons carry charge, energy, and entropy. The charge is -e. The entropy has to be calculated using statistical mechanics.

The internal energy per electron can be calculated using:

$$u = \frac{U}{N} = \int_{E_c}^{\infty} ED(E) f(E) dE$$

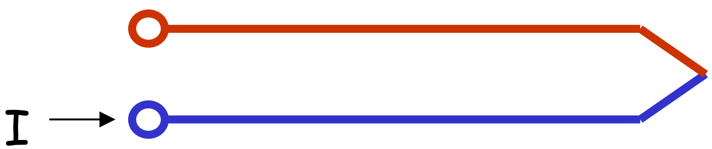
The energy current is related to the electrical current by the Peltier coefficient.

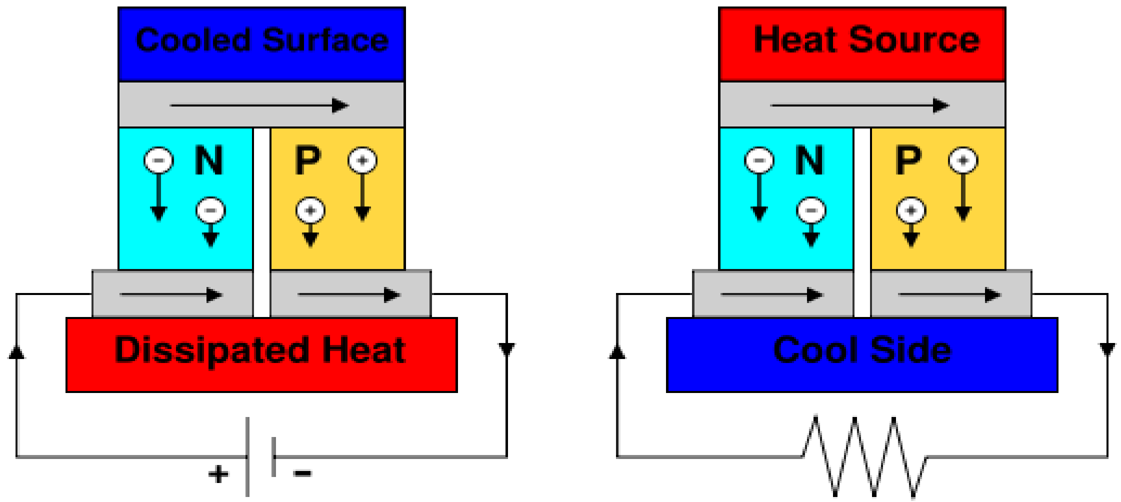
$$j_U = \Pi j_q$$

Π is the Peltier coefficient.

Thermoelectric effects

Peltier effect: driving a through a bimetallic junction causes heating or cooling.





Cooling takes place when the electrons make a transition from low entropy to high entropy at the junction.

http://en.wikipedia.org/wiki/Peltier_effect#Peltier_effect