Student Project: Physics of Semiconductor Devices

Exam Question: Effective mass - Exam October 2011

Q: *In a semiconductor where the bottom of the conduction band is at* $k = 0$*, an electron moves with a wave-number* $k = 10^9 m^{-1}$*.*

- *If the effective mass of electrons in the conduction band is* $0.5 \cdot m_e$*, what is the energy of this electron measured from the bottom of the conduction band?*
- *Electrons in the conduction band will occupy some states up until about* $k_B T$ *above the bottom of the conduction band. What is the maximum value of* k *that electrons would have from thermal fluctuations at room temperature? What consequences does this have for a light emitting diode?*

A: The energy-wavenumber, also called dispersion, relation in the vicinity of the bottom of the conduction band can be approximated by the free electron model. Which means that the energy has just a quadratic dependence on the wavenumber. For the one dimensional case it reads as:

$$E = \frac{\hbar^2}{2m^*} k^2. \quad (1)$$

Where the mass m has been replaced by the so called effective mass m^*. It is determined by the by the curvature of the bottom of the conduction band:

$$m^* = \frac{\hbar^2}{\frac{d^2E(k)}{dk^2}}. \quad (2)$$

This effective mass approximately includes the otherwise ignored influence of the lattice potential caused by the atomic nuclei.
For a k-value of $k = 10^9 m^{-1}$ and a effective mass of $0.5 \cdot m_e$ this yields an energy of $E = 0.0762 eV$.
Further physical constants needed for the calculation:

- reduced Planck constant $\hbar = 1.05457 \cdot 10^{-34} J \cdot s$
- electron mass $m_e = 9.1094 \cdot 10^{-31} kg$
- electron charge $e = 1.602 \cdot 10^{-19} C$

For the second question one inverts equation (1) to calculate the wave-number corresponding to $E_{th} = k_B T = 0.0253 eV$ at a room-temperature of $T = 292K$

$$k = \sqrt{\frac{2m^*}{\hbar^2} E}. \quad (3)$$

So at room temperature states up to $k_{th} = 5.76 \cdot 10^8 m^{-1}$ are occupied, see fig.(1).

Consequences for a light emitting diode:

At temperature $T = 0K$ a light emitting diode would only emit light with a frequency exactly corresponding to the band-gap energy $h\nu = E_g$, but as a consequence of these thermal fluctuations the spectral width of the emitted light broadens. The uncertainty of the frequency of the emitted light at room temperature is $\Delta\nu = k_b T/h = 2.41 \cdot 10^{14} Hz$. Or in term of wavelengths $\Delta\lambda = 1.24 \cdot 10^{-6} m = 124nm$, see fig.(2)

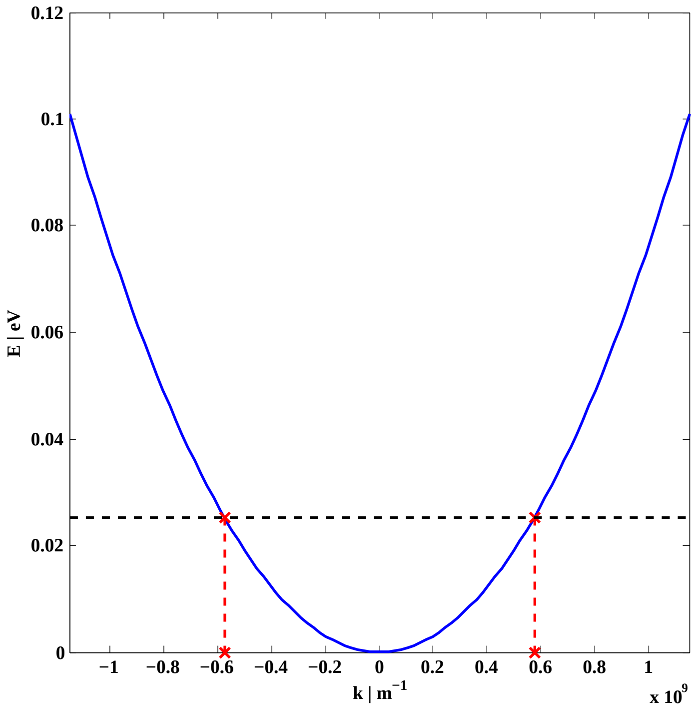

Figure 1: Energy as function of wave-number in the free electron approximation.

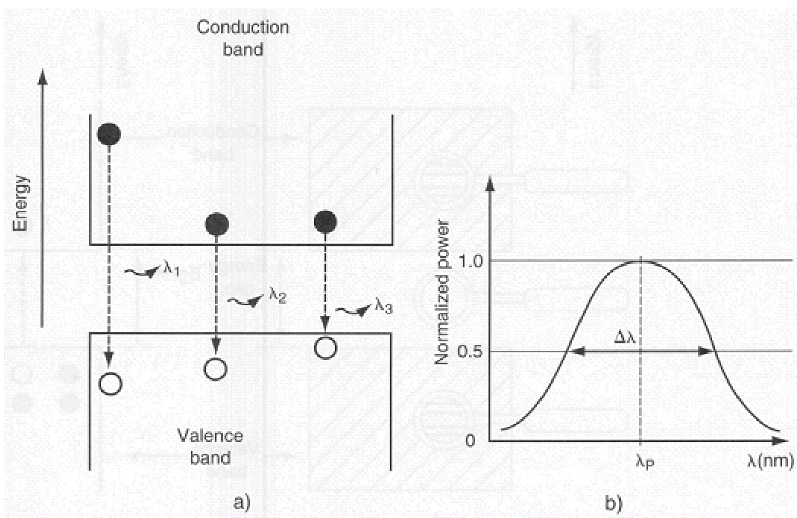

Figure 2: Broadening of the spectral width of a LED due to thermal fluctuations. Source:http://spie.org/x32442.xml