**Student Project: Physics of Semiconductor Devices**

**Exam Question: Effective mass - Exam October 2011**

**Q:** In a semiconductor where the bottom of the conduction band is at \( k = 0 \), an electron moves with a wave-number \( k = 10^9 \text{m}^{-1} \).

- If the effective mass of electrons in the conduction band is \( 0.5 \cdot m_e \), what is the energy of this electron measured from the bottom of the conduction band?

- Electrons in the conduction band will occupy some states up until about \( k_B T \) above the bottom of the conduction band. What is the maximum value of \( k \) that electrons would have from thermal fluctuations at room temperature? What consequences does this have for a light emitting diode?

**A:** The energy-wavenumber, also called dispersion, relation in the vicinity of the bottom of the conduction band can be approximated by the free electron model. Which means that the energy has just a quadratic dependence on the wavenumber. For the one dimensional case it reads as:

\[
E = \frac{\hbar^2}{2m^*} k^2. \tag{1}
\]

Where the mass \( m \) has been replaced by the so called effective mass \( m^* \). It is determined by the by the curvature of the bottom of the conduction band:

\[
m^* = \frac{\hbar^2}{2E(k)} \tag{2}
\]

This effective mass approximately includes the otherwise ignored influence of the lattice potential caused by the atomic nuclei.

For a \( k \)-value of \( k = 10^9 \text{m}^{-1} \) and an effective mass of \( 0.5 \cdot m_e \) this yields an energy of \( E = 0.0762 \text{eV} \).

Further physical constants needed for the calculation:

- reduced Planck constant \( \hbar = 1.05457 \cdot 10^{-34} \text{J} \cdot \text{s} \)
- electron mass \( m_e = 9.1094 \cdot 10^{-31} \text{kg} \)
- electron charge \( e = 1.602 \cdot 10^{-19} \text{C} \)

For the second question one inverts equation (1) to calculate the wave-number corresponding to \( E_{th} = k_B T = 0.0253 \text{eV} \) at a room-temperature of \( T = 292 \text{K} \),

\[
k = \sqrt{\frac{2m^*}{\hbar^2}E}. \tag{3}
\]

So at room temperature states up to \( k_{th} = 5.76 \cdot 10^8 \text{m}^{-1} \) are occupied, see fig.(1).

Consequences for a light emitting diode:

At temperature \( T = 0 \text{K} \) a light emitting diode would only emit light with a frequency exactly corresponding to the band-gap energy \( h\nu = E_g \), but as a consequence of these thermal fluctuations the spectral width of the emitted light broadens. The uncertainty of the frequency of the emitted light at room temperature is \( \Delta \nu = k_B T / h = 2.41 \cdot 10^{14} \text{Hz} \). Or in term of wavelengths \( \Delta \lambda = 1.24 \cdot 10^{-6} \text{m} = 124 \text{nm} \), see fig.(2)
Figure 1: Energy as function of wave-number in the free electron approximation.

Figure 2: Broadening of the spectral width of a LED due to thermal fluctuations. 
Source: http://spie.org/x32442.xml